Test case 2 : cone-sphere with a PEC patch

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Abstract

We deal with an alumina coated cone bounded by a sphere using a tangential connection. On a generator of the cone we conform a square patch made up of many rectangular pieces in a Perfectly Electric Conductor whose influence on RCS is analyzed through a frequency spectrum.

1 Technical formula for Fortran coded mesh

1.1 Coated sphere-cone

Let (x, y, z) be the coordinates so that the cone equation is (meaning z origin is taken at the sphere center z_1) :

$$\sqrt{x^2+y^2} \leq \left(\frac{R_{ext}}{\sin\alpha} - z\right)\tan\alpha$$

and the sphere equation is

$$x^2 + y^2 + z^2 \leq R_{ext}^2$$

The tangential connection is made testing for the location related to the line going through the origin and the tangential connection point located at $(x = R_{ext} \times \cos\alpha, z = R_{ext} \times \sin\alpha)$ of equation

$$z = \sqrt{x^2+y^2} \times \tan\alpha$$

If $z \geq \sqrt{x^2+y^2} \times \tan\alpha$ then we shall test against the cone function, otherwise against the sphere function.

The same functions are used with $R = R_{int}$ for determining the PEC reference below the material layer.

1.2 Patch build in rectangular coordinates

We request that the interval between patches is made up of at least two cells. So, if input coordinate x in one direction of the patch is relative to the patch in $[0 \ldots 1]$, let us define a local variable $x_l = n \times x - int(x \times n)$ defined in $[0 \ldots 1]$ where conductor test function is a trivial $|x_l - \frac{1}{2}| < \frac{f}{2}$ where f is the fraction of conductor on the patch per direction.

1.3 Patch conformation onto a cone

Let the cone equation be in cartesian coordinates $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ with origin at the top of the cone $(z_0 = 0)$:

$$x^2 + y^2 = z^2 \tan^2 \alpha$$

Let A be a point on the generator of coordinates $(x = h \tan \alpha, y = 0, z = h)$ from where to stick the patch orthogonally to the plane defined by $(A, \vec{\nu}, \vec{e}_2, \vec{\tau})$ where $\vec{\nu}$ is the local normal to the cone at point A, $\vec{\tau}$ is the tangent vector aligned with the generator. Let (X, Y, Z) be the coordinates in this referential. The curve we are looking for is defined by $Z = 0$ and $x^2 + y^2 = z^2 \tan^2 \alpha$ where

$$\begin{aligned}
&x = h \tan \alpha + X \cos \alpha + Z \sin \alpha = h \tan \alpha + X \cos \alpha \\
&y = Y \\
&z = h - X \sin \alpha + Z \cos \alpha = h - X \sin \alpha
\end{aligned}$$

so

$$(\cos^4 \alpha - \sin^4 \alpha) X^2 + 2h \sin \alpha X + Y^2 \cos^2 \alpha = 0$$

i.e.

$$\left(\frac{X - a}{a}\right)^2 + \left(\frac{Y}{b}\right)^2 = 1 \text{ with } a = h \frac{\sin \alpha}{\sqrt{\cos^2 \alpha - \sin^2 \alpha}} \text{ and } b = h \tan \alpha$$

Ellipticity factor e is defined and evaluates to:

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2} = \tan \alpha \ .$$

We then have to compute curvilinear abscissa along this ellipse. Let $M(\theta)$ be a point whose coordinates are parametrized by $X = a \sin \theta$ and $Y = b \cos \theta$ (where $a \geq b$), then curvilinear abscissa is given by (we can check that for small θ the value approximates to $a \times \theta$ since local radius at point $(X = 0, Y = b)$ is a) :

$$s(\theta) = a \int_0^\theta \sqrt{\left(1 - e^2 \sin^2(\tilde{\theta})\right)} \, d\tilde{\theta}$$

Figure 1 indicates how to geometrically compute the curvilinear abscissa. From an $M(X, Y)$ location, we compute the θ parameter as

$$\theta = atan(\frac{bX}{aY}) = atan2(b * X, a * Y)$$

and then derive the abscissa from $(a, 0)$ to $M(\theta)$ along the ellipse as :

$$S = s(\frac{\pi}{2}) - s(\theta)$$

where $s(\theta)$ is the elliptic function of the second kind. We compute it using function `elit` from [1].

References

[1] Shanjie Zhang and Jianming Jin. *Computation of Special Functions.* Wiley, 1996. LC: QA351.C45.

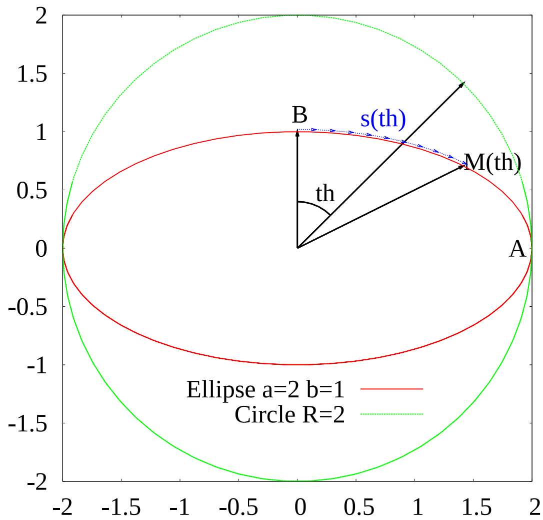

Figure 1: Curvilinear abscissa $s(\theta)$ along an ellipse arc between $B = (0, b)$ and $M(\theta)$ in (X, Y) plane.