Earth entry observations: Solution to inverse problem with incomplete data

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Abstract. While the Earth orbits the Sun, it is subject to impact by smaller objects ranging from tiny dust particles and space debris to much larger asteroids and comets. To study these collisions in more details, and to better understand phenomena during hypervelocity atmospheric entry, we present a practical algorithm connecting ground based observations with the properties of otherwise unknown objects entering the Earth's atmosphere. In particular, we derive analytical dependencies between space object mass, its size and other properties from the rate of body deceleration in the atmosphere.

Introduction

Techniques of determining the masses and bulk densities of a meteors and bolides have long been discussed in the literature dedicated to the meteor studies. These data are of current importance because of their implications for gaining information on cosmic-matter influx onto planets, for more reliably and quickly finding meteorites on the Earth's surface, for the studying of composition and structure of cometary nuclei, for calculating the orbital evolution timescale and the peak temperature reached on atmospheric entry. Majority of the existing conclusions in these applications are quite sensitive to assumed masses and bulk densities values. In turn these values received by means of various approaches sometimes differ from each other by more than on order. Our preference in such conditions should be based on correct physical models and the accessible experimental data, which allow avoiding rough assumptions and uncertainties in the explanation. Some approaches are inefficient if applied to studying large bodies for which the major part of the luminous segment of the trajectory corresponds to conditions of continuous-medium flow around the meteoroid (Gritsevich, 2008a). As one of the consequences, only a few meteorites successfully photographed by special cameras during their flight in the atmosphere were found on the ground using the existing data-processing techniques.

Dynamic and photometric estimates

The interpretation of the observations of meteors and fireballs is usually based on photometric methods or dynamical methods. The photometric methods use the fireball luminosity. It is usually assumed that a given fraction of the kinetic-energy of the body is converted into the visible radiation. The greatest uncertainty than is not well-known value of the luminous efficiency coefficient τ (Campbell-Brown, Koschny, 2004). The validity of the photometric approach in general usually is supported by the fact that the spectral lines of elements of most meteorites dominate in the meteor spectra. This suggests that the dominating contribution to the meteor luminosity comes from the emission of the material vaporized from the body surface. However, other important sources of emission have to be ignored.

The "dynamical" methods determine the body mass from the analysis of the observed drag in the atmosphere. The main drawback of these methods is in the necessity of the a priori assumption on the density and the shape factor of the body. These parameters currently cannot be directly obtained from the observations. The dynamical methods are often used if the falling of meteorites is expected. The mass of a fireball in the lower part of the trajectory is used to estimate the masses of the possible meteorites. The mass is usually directly calculated from the projection of the motion equation onto the tangent to the trajectory (see (Gritsevich, 2008b) for a review).

Discussion and Results

One of the basic results of the calculations presented in the table 1 is the fact that the extra-atmospheric masses determined from the braking intensity over the entire observed section of the trajectory significantly differ from the masses based on the glow intensity of the fireballs (i.e., photometric masses). This conclusion remains even for assumption of quite fragile meteor bodies with the bulk density comparable to the density of ice. Halliday et al. (1996) used photometric formula with modified luminous efficiency coefficient τ which has to be dependant on the fireball velocity. More general approach for studying the radiation of a fireball was proposed by Gritsevich and Koschny, 2011. Their study takes recent results in fireball aerodynamics and considers them together with the classical postulate that a fraction of the meteoroid kinetic energy is transformed into radiation during its flight. This gives us a new analytical dependence, which in particular shows that the fireball luminosity in general is proportional to the body pre-entry mass value, its initial velocity to the power of 3, and the sine of the slope between horizon and trajectory.

Of course, most preferable are models in which all used parameters or at least a range of their possible values are known for sure. Dynamic methods of determination of parameters of a meteor bodies for today are well established to be used with good accuracy. Additionally we have better and fuller knowledge of the forms of meteorites, and their bulk densities. Drag coefficients values for the bodies of meteorite-like shapes are specified (see, e.g. Zhdan et al., 2007). It is notable that some properties of meteoroid types that are too fragile to survive during atmospheric entry to become meteorites can be analyzed and collected in the form of interplanetary dust. Therefore gathering of these particles and development of the techniques on studying their structure and properties has today a great importance and would complement this study.

Conclusions

The applied general approach helps in more precise understanding of the extensive observational data obtained by the fireball networks. During our data processing we discovered several sufficiently thermostable meteor bodies whose mass loss parameters were almost zero. Based on this fact, we conclude that the dominant contribution to the luminosity of such meteor body is made by the radiation of the atmospheric gas in the shock layer near the body rather than by its evaporation. This conclusion corresponds to the direct calculations of intensity of luminosity of the heat atmospheric gas near the meteor body (Gritsevich and Stulov, 2007).

The approximation of the actual data using theoretical models in general makes it possible to achieve additional estimates, which do not directly follow from the observations. In other words, the correct mathematical modeling of meteor events in the atmosphere is necessary for further estimates of the key parameters, including the extra-atmospheric mass, the ablation coefficient, and the effective enthalpy of evaporation. This information is used to answer important questions, including:

How to specify and speed up the recovery of a recently fallen meteorite, not letting weathering to affect samples too much?

How entering object affects Earth's atmosphere and (if applicable) Earth's surface?

How to predict these impact consequences based on atmospheric trajectory data?

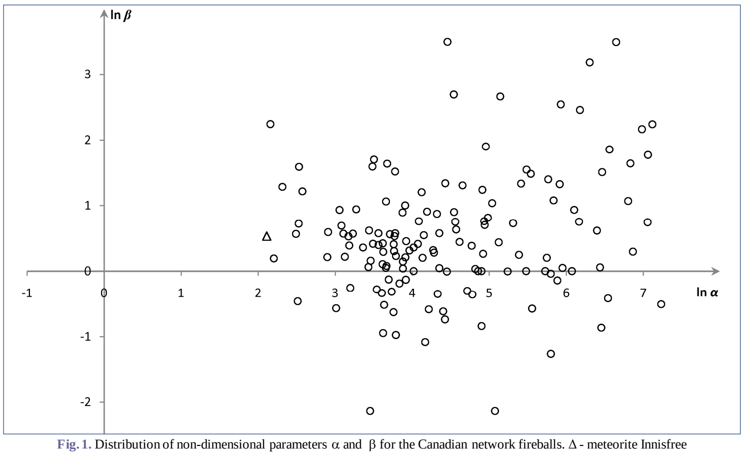

Fig. 1. Distribution of non-dimensional parameters α and β for the Canadian network fireballs. Δ - meteorite Innisfree

fireball №	V_e, km/s	α	β	$10^2\times\sigma$	M, g	M_{ph}, g	M_{ph}/M_s
878	15.5	58.7	1.51	1.26	109	3000	27.5
1010	18.4	71.58	1.38	0.81	66	3100	46.8
207	17.9	24.45	0.78	0.48	1223	3400	2.8
299	23.6	48.36	1.04	0.37	607	3400	5.6
303	14.1	44.15	1.26	1.27	685	3500	5.1
307	21	12.08	1.76	0.8	27852	3500	0.1
852	15.9	21.27	2.54	2.01	1678	3600	2.1
996	26.9	59.56	2.13	0.59	138	3700	26.8
231	27.9	52.72	1.37	0.35	1717	4300	2.5
888	25.5	31.85	1.17	0.36	1942	4300	2.2

Tab. 1. The computed initial mass values for the Canadian network fireballs

Mathematical model

The physical problem of the meteor body deceleration in the atmosphere has been considered in the number of papers and monographs, see e.g. (Stulov et al, 1995). The classical dynamic third-order system has been constructed, where the body mass M (t), its height above the planetary surface h (t) and its velocity V (t) are the phase variables. The equations of motion projected onto the tangent and to the normal to the trajectory appear as

$$M\frac{dV}{dt} = -D + P\sin\gamma, \tag{1}$$

$$MV\frac{d\gamma}{dt} = P\cos\gamma - \frac{MV^2}{R}\cos\gamma - L, \tag{2}$$

$$\frac{dh}{dt} = -V\sin\gamma, \tag{3}$$

where $D = c_d\rho_a V^2 S/2$ is the drag force, $L = c_L\rho_a V^2 S/2$ is the lifting force, and $P = Mg$ is the body weight. Here M and V are the body mass and velocity, respectively; t is the time; h is the height above the planetary surface; γ is the local angle between the trajectory and the horizon, S is the area of the middle section of the body; ρ_a is the atmospheric density, g is the acceleration due to gravity; R is the planetary radius, and c_d, c_L are the drag coefficient and the lift coefficient, respectively.

Equations (1)–(3) are complemented by the equation for the variable mass of the body:

$$H^*\frac{dM}{dt} = -\frac{1}{2}c_h\rho_a V^3 S \tag{4}$$

where H^* is the effective enthalpy of destruction and c_h is the coefficient of heat exchange. It is assumed that the entire heat flux from the ambient gas is spent to the evaporation of the surface body material. Using Eq. (3), it is possible to introduce a new variable h and pass to convenient dimensionless quantities $M = M_e m$, $V = V_e v$, $h = h_0 y$, $\rho_a = \rho_0\rho$, $S = S_e s$,(where h_0 is the height of the homogeneous atmosphere, ρ_0 is the atmospheric density near the planetary surface, and the subscript e indicates the parameters at the entry to the atmosphere). Since the velocities in the problem under consideration are high enough (in the range from 11 to 72 km/s), the object weight in Eq. (1) is conventionally neglected. Variations in a slope γ are not significant and usually they are not taken into account. With allowance for the above considerations, the equations for calculating the trajectory eventually acquire the following simple form:

$$m\frac{dv}{dy} = \frac{1}{2}c_d\frac{\rho_0 h_0 S_e}{M_e}\frac{\rho v s}{\sin\gamma},\quad \frac{dm}{dy} = \frac{1}{2}c_h\frac{\rho_0 h_0 S_e}{M_e}\frac{V_e^2}{H^*}\frac{\rho v^2 s}{\sin\gamma} \tag{5}$$

To find the analytical solution of Eqs. (5), we suggest that the atmosphere is isothermal: $\rho = e^{-y}$. According to B. Yu. Levin (1956) we also assume that the middle section and the mass of the body are connected by the following relation $s = m^{\mu}$, $\mu = const$. The parameter μ characterizes the possible role of rotation during the flight.

Then the solution of Eqs. (5) with the initial conditions $y = \infty$, $v = 1$, $m = 1$ has the form

$$m = \exp\left(-\frac{\beta}{1-\mu}(1-v^2)\right), \tag{6}$$

$$y = \ln\alpha + \beta - \ln\frac{\Delta}{2},\ \Delta = Ei(\beta) - Ei(\beta v^2),\ Ei(x) = \int_{-\infty}^{x}\frac{e^z dz}{z}, \tag{7}$$

$$\alpha = \frac{1}{2}c_d\frac{\rho_0 h_0 S_e}{M_e\sin\gamma}, \tag{8}$$

$$\beta = (1-\mu)\frac{c_h V_e^2}{2c_d H^*} \tag{9}$$

where α is the ballistic coefficient and β is the mass loss parameter.

Hereafter let us use the analytical solution (7) as the general theoretic height-velocity relation.

The values of the parameters α and β providing for the best fit of the observed physical process can be found by the method proposed by (Gritsevich, 2009). The sum of the squared deviations of the actually observed altitudes h_i and velocities V_i of motion at certain points i = 1, 2, …, n of the desired curve described by Eq. (7) from the corresponding values e^{-y} calculated using Eq. (7) is used as the fitting criterion. Then the desired parameters are unambiguously determined by the following formulas:

$$\alpha = \frac{\sum_{i=1}^{n} e^{-\beta-y_i}\cdot\Delta_i}{2\sum_{i=1}^{n} e^{-2y_i}}, \tag{10}$$

$$\sum_{i=1}^{n}\left[\left(\Delta_i\sum_{i=1}^{n}\exp(-2y_i) - \left(\sum_{i=1}^{n}\Delta_i\exp(-y_i)\right)\exp(-y_i)\right)\cdot\left(\Delta_i - (\Delta_i)'_\beta\right)\right] = 0, \tag{11}$$

$$\frac{\sum_{i=1}^{n} e^{-2y_i}\sum_{i=1}^{n}\left(((\Delta_i)'_\beta - \Delta_i)^2 + (\Delta_i - 2\alpha\exp(\beta - y_i))((\Delta_i)''_\beta - 2(\Delta_i)'_\beta + \Delta_i\right)}{\left(\sum_{i=1}^{n}\exp(-y_i)(\Delta_i - (\Delta_i)'_\beta\right)^2} > 1 \tag{12}$$

Here $v_i = V_i / V_e$, $y_i = h_i / h_0$, $\Delta_i = Ei(\beta) - Ei(\beta v_i^2)$, $(\Delta_i)'_\beta = d\Delta_i / d\beta$, $(\Delta_i)''_\beta = d^2\Delta_i / d\beta^2$.

It is essentially important to note, that the value of β specified by Eqs. (10)-(12) describes the mass-loss efficiency along the entire studied segment of the meteor trajectory due to both evaporation and melting of the outer layer followed by blowing-off of the liquid film by the flow and detachment of secondary-size fragments from the parent body (Gritsevich, 2008c).

The obtained parameters are used to calculate the mass of a meteor body. In particular, the initial mass M_e can be estimated using the value of ballistic coefficient α in the following way:

$$M_e = \left(\frac{c_d A_e}{2}\frac{\rho_0 h_0}{\alpha\sin\gamma}\right)^3\cdot\rho_b^{-2}$$

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