Multidisciplinary Design Optimization of Aircraft Configurations Part 1: A modular coupled adjoint approach



Once numerical simulations are developed, they can be used for design optimization



Design optimization problem: minimizef(x)objectivewith respect toxdesign variablessubject to $c(x) \le 0$ constraints

Aircraft are performance critical and hence numerical optimization is especially valuable

JAL B787 climbing after takeoff from SAN • © 2013 J.R.R.A. Martins

Complex systems require the consideration of multiple disciplines, hence MDO was born



Multidisciplinary Design Optimization of Aircraft Configurations Part 1: A modular coupled adjoint approach

- A review of MDO architectures
- A new MDO architecture
- Applications

Multidisciplinary Design Optimization of Aircraft Configurations Part 1: A modular coupled adjoint approach

A review of MDO architectures

Sequential optimization is equivalent to coordinate descent



Sequential optimization fails to find the multidisciplinary optimum



[Chittick and Martins, Struct. Multidiscip. O., 2008]

The design process in industry is not tailored towards MDO



Personnel hierarchy



Design process

The N2 Diagram and Design Structure Matrix (DSM)



- Components on main diagonal, coupling data on off-diagonal nodes
- Component inputs in same column, component outputs in same row
- External inputs and outputs may also be included



XDSM http://mdolab.engin.umich.edu/content/xdsm-overview



The XDSM (eXtended Design Structure Matrix) is a tool used to visualize MDO processes. It is an extension of the classical Design Structure Matrix commonly used in systems engineering to describe the interfaces among components of a complex system. In a computational MDO context, the complex system is the MDO architecture, the components of the system are pieces of software (disciplinary analyses, optimization algorithms, surrogate models, etc.) used by the architecture, and the interfaces between components are the data exchanged by this software. Because the architecture also contains an algorithm defining the order in which the software is run, a numbering system and lines depicting the process are introduced in the diagram. In this way, we are able to capture all of the data and process flow of an architecture in a single diagram.

The full details of how to construct and interpret XDSMs are the subject of the paper cited in the foctnote.4

For those interested in constructing XDSMs for their own work, see the attached files. We draw our diagrams using the TikZ package in LaTeX. The files contain the specific block and line styles, TikZ library imports, and formats that are common to all of our diagrams. We have also included some example diagrams and a how-to guide for the LaTeX files. Comments and suggestions are welcome.

A Python script for automatically generating XDSM tex sources has been added. This script contains a class to which components and dependencies can be added, and this class automatically writes a tex file that draws the diagonal and off-diagonal blocks, as well as data flow lines. Details can be found in the Python script.

A. B. Lambe and J. R. R. A. Martins, "Extensions to the Design Structure Matrix for the Description of Multidisciplinary Design, Analysis, and Optimization Processes", Structural and Multidisciplinary Optimization, vol. 46, no. 2, p. 273-284, 2012.

Attachment	Size
🗎 diagram_border.tex	437 bytes
🛅 diagram_styles.tex	4.81 KB
C XDSM_how_to.txt	5.16 KB
🖹 co.tex	3.07 KB
CO.pdf	59.36 KB
MDF.tex	3.04 KB
MDF.pdf	57.76 KB
C XDSM.py.txt	4.83 KB

A Jacobi Multidisciplinary Analysis Example



An Optimization Problem



- Follow sequence of numbers and thin black lines
- When number or index is repeated, procedures can be parallelized
- Close the loops

The Multidisciplinary Feasible (MDF) Architecture

Problem Formulation

 $\begin{array}{ll} \text{minimize} & f_0\left(x, y\left(x\right)\right) \\ \text{with respect to} & x \\ \text{subject to} & c_0\left(x, y\left(x\right)\right) \ge 0 \\ & c_i\left(x_0, x_i, y_i\left(x_0, x_i, y_{j \neq i}\right)\right) \ge 0 \end{array}$

The Multidisciplinary Feasible (MDF) Architecture



Individual Discipline Feasible (IDF)

Problem Formulation

 $\begin{array}{ll} \text{minimize} & f_0\left(x, y\left(x, y^t\right)\right) \\ \text{with respect to} & x, y^t \\ \text{subject to} & c_0\left(x, y\left(x, y^t\right)\right) \ge 0 \\ & c_i\left(x_0, x_i, y_i\left(x_0, x_i, y_{j\neq i}^t\right)\right) \ge 0 \\ & y_i^t - y_i\left(x_0, x_i, y_{j\neq i}^t\right) = 0 \end{array}$

Individual Discipline Feasible (IDF)



that can be run in parallel



[Martins and Lambe, "MDO: A Survey of Architectures", AIAAJ, 2013]

Benchmarking MDO Architectures



[Tedford and Martins, *Optimization and Engineering*, 2010]

Multidisciplinary Design Optimization of Aircraft Configurations Part 1: A modular coupled adjoint approach

MAUD—Modular Analysis and Unified Derivatives

3 major challenges



 Computational costly to evaluate objective and constraints

2. Multiple highly coupled systems



3. Large numbers of design variables, design points and constraints

Gradient-based methods take a more direct path to the optimum



Gradient-based optimization is the only hope for large numbers of design variables



Gradient-based optimization works best with accurate and efficient gradient computations



Gradient-based optimization requires gradient of objective and Jacobian of constraints

$$egin{aligned} & \min_{x \in \mathbb{R}^n} & f(x,y(x)) \ & ext{s.t.} & h(x,y(x)) = 0 \ & g(x,y(x)) \leq 0 \end{aligned}$$

x: design variables

y: state variables, determined implicitly by solving R(x, y(x)) = 0

Need df/dx (and also dh/dx, dg/dx).

Methods for computing derivatives

Monolithic Black boxes input and outputs	Finite-differences	$\frac{\mathrm{d}f}{\mathrm{d}x_j} = \frac{f(x_j + h) - f(x)}{h} + \mathcal{O}(h)$
	Complex-step	$\frac{\mathrm{d}f}{\mathrm{d}x_j} = \frac{\mathrm{Im}\left[f(x_j + ih)\right]}{h} + \mathcal{O}(h^2)$
Analytic Governing eqns state variables	Direct Adjoint	$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{\partial f}{\partial x} - \frac{\partial f}{\partial y} \begin{bmatrix} \partial R \\ \partial y \end{bmatrix}^{-1} \frac{\partial R}{\partial x}$
Algorithmic differentiation <i>Lines of code</i> <i>code variables</i>	Forward $\begin{bmatrix} 1 & 0 & \dots & 0 \\ -\frac{\partial T_2}{\partial t_1} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ -\frac{\partial T_n}{\partial t_1} & \dots & -\frac{\partial T_n}{\partial t_{n-1}} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & \cdots & 0 \\ \frac{\mathrm{d}t_2}{\mathrm{d}t_1} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \frac{\mathrm{d}t_n}{\mathrm{d}t_1} & \cdots & \frac{\mathrm{d}t_n}{\mathrm{d}t_{n-1}} \end{bmatrix} = \mathbf{I} = \begin{bmatrix} 1 - \frac{\partial T_2}{\partial t_1} \cdots & -\frac{\partial T_n}{\partial t_1} \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\frac{\partial T_n}{\partial t_{n-1}} \\ 0 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \frac{\mathrm{d}t_2}{\mathrm{d}t_1} \cdots \frac{\mathrm{d}t_n}{\mathrm{d}t_1} \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & 1 & \frac{\mathrm{d}t_n}{\mathrm{d}t_{n-1}} \\ 0 & \cdots & 0 & 1 \end{bmatrix}$

[Martins and Hwang, AIAA Journal, 2013]

[Martins et al., ACM TOMS, 2003]

Analytic methods evaluate derivatives by linearizing the governing equations

Need df/dx (and also dh/dx, dg/dx), f(x, y(x))

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x}$$

Derivative of the governing equations: R(x, y(x)) = 0

$$\frac{\mathrm{d}R}{\mathrm{d}x} = \frac{\partial R}{\partial x} + \frac{\partial R}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \quad \Rightarrow \quad \frac{\partial R}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{\partial R}{\partial x}$$

Substitute result into the derivative equation

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{\partial f}{\partial x} - \frac{\partial f}{\partial y} \left[\frac{\partial R}{\partial y}\right]^{-1} \frac{\partial R}{\partial x}$$

$$\psi$$

Cost of adjoint evaluation is independent of the number of design variables





Coupled solution of aerodynamics and structures, and the corresponding coupled adjoint

Solve the coupled governing equations

$$R(x, y) = \begin{bmatrix} R_A(x, y_A, y_S) \\ R_S(x, y_A, y_S) \end{bmatrix} = 0$$

 $\Gamma_{\Omega} \sigma T \sigma T$

form and solve the adjoint equations

 $(\mathbf{I}\mathbf{X})$

$$\frac{\partial R}{\partial y}^{T}\psi = \frac{\partial f}{\partial y}^{T} \text{ where } \frac{\partial R}{\partial y}^{T} = \begin{bmatrix} \frac{\partial R_{A}}{\partial y_{A}} & \frac{\partial R_{S}}{\partial y_{A}} \\ \frac{\partial R_{A}}{\partial y_{S}} & \frac{\partial R_{S}}{\partial y_{S}}^{T} \end{bmatrix}$$
and compute the gradient
$$\frac{df}{dy} = \frac{\partial f}{\partial y} - \psi^{T}\frac{\partial R}{\partial y}$$

OX

ØХ

The coupled adjoint approach has shown promising results for a 2-discipline high-fidelity problem



[Kenway, Kennedy and Martins, AIAA Aviation 2014]

The adjoint method computes the gradient at the cost of about 1 evaluation



= O(100) iterations x O(1) evaluation = O(100) hrs

Gradient-based methods enable large-scale optimization but they are difficult to implement



Disadvantages:

- The adjoint method requires modification of the computational model
- The entire model must be differentiable

Our objective: Facilitate the coupling of disciplines and the coupled adjoint method implementation



Outline



For optimization to be a useful design tool, setting up should be as streamlined as possible

Run optimization

Interpret results

Set up a new optimization problem

- Change the objective function
- Add/remove design variables from the inputs
- Add/remove constraints from the outputs
- Add/remove disciplines and models

We need a software framework to enable a modular approach
However, existing software frameworks are not designed to support the coupled adjoint method



Our approach is to formulate computational models as a system of equations

Any computational models can be decomposed into

Input variables $x = (x_1, \ldots, x_m)^T$ State variables $y = (y_1, \ldots, y_p)^T$ Output variables $f = (f_1, \ldots, f_q)^T$

The objective is to express it as a system of algebraic equations,

 $\langle \rangle$

$$R(u) = 0$$
 where $u = \begin{pmatrix} x \\ y \\ f \end{pmatrix}$



Example: aerodynamic and structural analysis coupled together

Question: how do we compute *df/db* in the following problem?

Variable	Туре		
Wing span	input	b :	$b = b^*$
Pressures	state	<i>p</i> :	A(b, p, d) = 0
Displacements	state	<i>d</i> :	d = S(b, p)
Lift	output	<i>f</i> :	f = F(b, p)

Formulated as a system of algebraic equations:

$$u = \begin{bmatrix} b \\ p \\ d \\ f \end{bmatrix} \quad R(u) = \begin{bmatrix} R_b(b, p, d, f) \\ R_p(b, p, d, f) \\ R_d(b, p, d, f) \\ R_f(b, p, d, f) \end{bmatrix} = \begin{bmatrix} b - b^* \\ A(b, p, d) \\ d - S(b, p) \\ f - F(b, p) \end{bmatrix}$$

We derived the unifying chain rule, which generalizes all differentiation methods



[Hwang and Martins, AIAAJ, 2013]

This equation inspired the development of the Modular Analysis and Unified Derivatives (MAUD) framework

The MAUD framework was developed to facilitate two tasks:

- 1. Solution of a computational model with multiple components
- 2. Efficient computation of the derivatives of the coupled system of computational models

The MAUD architecture works by viewing the model as one large nonlinear system



MAUD

The MAUD framework core solves 4 types of sub-problems

- 1. Nonlinear system
- 2. Newton system
- 3. Derivatives in forward mode
- 4. Derivatives in reverse mode

$$R(u) = 0$$

$$\frac{\partial R}{\partial u}\Delta u = -r$$

 $\frac{\partial R}{\partial u}\frac{\mathrm{d} u}{\mathrm{d} r} = \mathcal{I}$

$$\frac{\partial R}{\partial u}^T \frac{\mathrm{d} u}{\mathrm{d} r}^T = \mathcal{I}$$

MAUD uses linear ar take advantage of the p





MAUD requires user to define a few basic functions for each component

		System classes			
		System	Compound System	E lementary System	
hods	$apply_nonlinear$		Recursive	User-implemented	
	$apply_linear$		Recursive	User-implemented or FD^*	
met	$solve_nonlinear$	Newton with	Nonlinear block	Optional	
Virtual 1		line search	Gauss–Seidel/Jacobi		
	$solve_linear$	Krylov-type with	Linear block	Optional	
		preconditioning	Gauss-Seidel/Jacobi		

*FD: finite-difference approximation of the Jacobian.

The user provides the computation of partial derivatives, and MAUD computes the coupled derivatives automatically

$$\frac{\partial R}{\partial u}\frac{du}{dr} = \mathcal{I} = \left[\frac{\partial R}{\partial u}\right]^T \left[\frac{du}{dr}\right]^T$$



MAUD uses a multiple component hierarchical representation to handle multi-physics systems



This hierarchical component representation enables better parallelization and sequencing



MAUD now provides the core algorithms in the OpenMDAO framework



- Originally developed by team at NASA Glenn
- Python-based, open source framework for coupling multiple models and optimization
- Facilitates collaboration between industry, academia, and government
- Provides a common platform for the development of new multidisciplinary analysis and design methods

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Applications of MAUD

A simple application of MAUD: Low-fidelity wing aerostructural analysis and design



- Lifting line aerodynamic analysis
- Spatial beam element structural analysis
- Coupled solution yields wing flying shape

Breaking down the problem into multiple components is beneficial in MAUD



- Easier to debug
- Component derivatives are easier
- More flexibility in algorithms and parallelization

Problem structure can be visualized using an interactive design structure matrix diagram



- Hierarchy of the problem is clearly visible
- The structure is shown as a direct structure matrix
- Couplings are shown when hovering on components
- This facilitates the exploitation of problem structure

The results can be visualized in real time



A more complex application of MAUD: Aircraft design with allocation, and mission optimization



[Hwang and Martins, AIAA 2016-1662]

The current aircraft design optimization approach is multipoint fuel burn minimization





Select a single design mission range, R

Select points $(M_1, C_{L,1}) \dots (M_n, C_{L,n})$ and their weights $w_1 \dots w_n$

minimize w_1 fb(R, M₁, C_{L,1}) + ... + w_n fb(R, M_n, C_{L,n})

with respect to the aircraft design



In reality, the allocation, mission profiles, and design should be optimized simultaneously

maximize airline profit with respect to design: shape, structure, engine

mission: Mach number, altitude profile

airline allocation: flights per day

Why?

- Aircraft often flown below their range
- Climb/descent significant on short flights
- Model morphing and continuous descent
- Determine optimal cruise Mach numbers



We perform the allocation-mission-design optimization of an aircraft in a 128-route network

Linearly tapered wing with Euler analysis twist and shape design variables

128 route network altitude profiles and optimal cruise M design variables





Potential issue: Need to compute the performance of aircraft in a 128-route network



The solution is to replace the CFD with a dynamically re-trained surrogate model



The allocation-mission-design optimization involves O(1000) design variables







O(100) Aircraft geometric variables O(1000) Mission profile variables O(100) Airline allocation variables

We parametrize a B717-based wing with shape and twist variables



Free-form deformation

We use CFD to solve the Euler equations



SUMad flow solver

We developed a unique mission analysis tool within the parallel framework



The framework automatically computes derivatives using the adjoint method

The mission analysis solves the flight equilibrium equations

$$L + W\cos\gamma - T\sin\alpha + \frac{W}{g}v^2\cos\gamma\frac{\mathrm{d}\gamma}{\mathrm{d}x} = 0$$



The altitude and Mach profiles can be optimized using a B-spline parametrization



Multiple trajectories can be optimized quickly



[Kao, Hwang, Martins, Gray, and Moore, AIAA 2015-0136]

Allocation problem: Maximize airline profit for a given fleet and network



The optimization problem contains over 6000 design variables

	Variable	Quantity
maximize	profit	
with respect to	to twist	
	shape	72
	cruise Mach number for each route (between 0.6 and 0.82)	
	altitude control points for each route	
	passengers per flight for each aircraft type and route	
	flights per day for each aircraft type and route	
	Total number of design variables	6061
subject to	ubject to wing volume constraint	
	wing thickness constraints	100
	idle thrust KS constraint for each route	128
	max thrust KS constraint for each route	128
	linear climb angle bounds for each mission	22875
	demand constraint for each route	128
	total flight time constraint for each aircraft	5
	Total number of constraints	23365

Allocation-mission-design optimization yields a 27% increase in airline profit



For the AMD optimization, the next-generation aircraft is flown more on the short range routes



Despite the 6000 design variables, the optimizer uses only ~100 iterations


The AMD-optimized wing shape is different from the multipoint result



Multipoint-optimized (minimize fuel burn) AMD-optimized (maximize profit)



Summary





- Introduced MDO architectures
- Developed MAUD, a novel algorithmic framework for coupled analysis and gradient computation
- MAUD demonstrated in large-scale MDO problems, including high-fidelity models
- Implemented MAUD architecture in OpenMDAO

http://mdolab.engin.umich.edu/publications

John Hwang



Go forth and optimize!



MDOIab Newsletter-Fall 2014



Dear Friend,

Welcome to the MDOlab newsletter, an update on research and open source software that we send a few times a year. You are receiving this because I think you are interested in numerical optimization, MDO, engineering design, or aircraft design. If this is not the case, feel free to <u>unsubscribe</u>. If you know someone who might like to subscribe, please forward them this newsletter. Best regards.



Latest publications

Wing aerodynamic shape optimization benchmark



The AIAA <u>Aerodynamic Design Optimization Discussion</u> <u>Group</u> developed a series of benchmark cases. In this paper, we solve the RANS-based wing optimization problem, iry to find multiple local minima, and solve a number of related wing design optimization problems. The initial and optimized geometries and meshes are <u>provided here</u>.

[Paper] [Preprint] [Optimization movie]

Aerodynamic design optimization of a blended-wing body aircraft



This builds on our previous work on <u>stability-constrained flying</u> <u>wing optimization</u>. A series of RANS-based aerodynamic design optimization studies shows the tradeoffs between drag, trim, and stability for the NASA/Boeing BWB. The photo on the left shows SD-printed models with pressure colomaps.

[Paper] [Preprint]

Satellite multidisciplinary design optimization benchmark



In collaboration with NASA and the <u>Michigan Exploration</u> <u>Lab</u>, we developed a new large-scale benchmark MDO problem, and solved a problem with 25,000 design variables and 2.2 million state variables by optimizing the data downloaded from a CubeSat subject to operational and physical constraints. This problem is now a <u>plugin</u> in the <u>OpenMDAO</u> open source project.





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Selected publications

- J. R. R. A. Martins and J. T. Hwang. Review and unification of methods for computing derivatives of multidisciplinary computational models. AIAA Journal, 51(11):2582–2599, November 2013. doi: 10.2514/1.J052184.
- 2. J. R. R. A. Martins and A. B. Lambe. Multidisciplinary design optimization: A survey of architectures. AIAA Journal, 51(9):2049–2075, September 2013. doi:10.2514/1.J051895.
- J. T. Hwang, D. Y. Lee, J. W. Cutler, and J. R. R. A. Martins. Large-scale multidisciplinary optimization of a small satellite's design and operation. Journal of Spacecraft and Rockets, 51(5):1648–1663, September 2014. doi: 10.2514/1.A32751.
- 4. G. K. W. Kenway and J. R. R. A. Martins. Multipoint high-fidelity aerostructural optimization of a transport aircraft configuration. Journal of Aircraft, 51(1):144–160, January 2014. doi:10.2514/1.C032150.
- G. K. W. Kenway, G. J. Kennedy, and J. R. R. A. Martins. Scalable parallel approach for high-fidelity steadystate aeroelastic analysis and derivative computations. AIAA Journal, 52(5):935–951, May 2014. doi: 10.2514/1.J052255.
- R. E. Perez, P. W. Jansen, and J. R. R. A. Martins. pyOpt: a Python-based object-oriented framework for nonlinear constrained optimization. Structural and Multidisciplinary Optimization, 45(1):101–118, January 2012. doi:10.1007/s00158-011-0666-3.
- J. T. Hwang, S. Roy, J. Y. Kao, J. R. R. A. Martins, and W. A. Crossley. Simultaneous aircraft allocation and mission optimization using a modular adjoint approach. In Proceedings of the 56th AIAA/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Kissimmee, FL, Jan. 2015. AIAA 2015-0900.
- 8. J. Y. Kao, J. T. Hwang, J. R. R. A. Martins, J. S. Gray, and K. T. Moore. A modular adjoint approach to aircraft mission analysis and optimization. In Proceedings of the AIAA Science and Technology Forum and Exposition (SciTech), Kissimmee, FL, January 2015. AIAA 2015-0136.