Some New Directions in Computational Electromagnetics for Solving Real-World problems

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Galaxy Assembly & Evolution

SKA: atomic gas, star formation, feedback

ALMA: molecular gas, star formation

JWST: dust, star formation

Optical/UV: stars, star formation

X- and γ-rays: feedback
Astrobiology

SKA: protoplanetary disks, molecules, planets, SETI

ALMA: protoplanetary disks, molecules

JWST: protoplanetary disks

Optical: protoplanetary disks, planets
Conventional MoM Limitations

Electrically large objects

- Long execution time
- Huge memory requirement
- Inefficient frequency analysis
- Inefficient Parametric analysis
Miniature Antenna Technologies for Future NASA Exploration Missions

**Description and Objectives:**
- Develop new design concepts and candidate miniature antenna structures capable of supporting the communication needs of future Lunar and Martian surface exploration activities.
- Develop compact, self-powering, self-oscillating communications package utilizing miniature antenna development effort.
- Perform trade-off studies among in-house miniature antenna designs and state-of-the-art commercial off-the-shelf (COTS) antennas for Exploration Missions.
- Develop processing algorithm for a randomly distributed network of Lunar surface sensors to enable a surface-to-orbit communication without the need of a Lunar surface base station.

**Application: Lunar Surface Exploration Missions**
- Robots and Rovers
- Surface Sensors/Probes
- Astronaut EVA
- Nanosatellites

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**Technology Products:**
- Folded Hilbert Curve Fractal Antenna
- Compact Microstrip Monopole Antenna
- Solar Cell Integrated Antenna
- Miniaturized antenna for Bio-MEMS Sensors
- Two-layer Sector Miniature Antenna
- MEMS Integrated Reconfigurable Antenna

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**Sensor Web Interconnections**
- Randomly located antenna/sensors
- Random Sensor Network Array

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**Technology Readiness Levels (TRL):**
- TRL_{in} = 2
- TRL_{out} = 3
Gain Pattern: Front Card Location

Card Alone

Card + Laptop

Card + Laptop + User

Total Gain

10 dB

5 dB

0 dB

-5 dB

-10 dB

-15 dB
Fig. 9. (a) Optical image and (b) 100–112 GHz wide-band holographic image of a clothed mannequin with a concealed Glock-17 handgun.

Fig. 10. Current wide-band holographic millimeter-wave imaging system.
CLOAKING—HOW PROMISING?
SMALL ANTENNA
Effect of the components

- Limited available volume
- Circuits and components
- Antenna: only component with physical limitations for miniaturisation!
Applications

• Electronic Toll Collection
• Access Control
• Animal Tracking
• Inventory Control
• Tracking Runners in Races!
Inductive Coupling: the author, Amal Graafstra, and his girlfriend, Jennifer Tomblin, have matching RFID implants.
The VeriChip implantable RFID tag, shown below, is the only tag approved for use in humans for a medical application. It is a simple device consisting of a coil of wire and a hermetically sealed microchip within a glass capsule. The coil acts as an antenna and uses an RFID reader’s varying magnetic field to power the microchip and transmit a radio signal. Each VeriChip’s signal is a unique identifying number that links to a medical record database.

**SIZE**
The device is 11 millimeters long and about 1 mm in diameter, comparable to a grain of rice.

**TISSUE-BONDING CAP**
A cap made from a special plastic covers a hermetically sealed glass capsule containing the RFID circuitry. The plastic is designed to bond with human tissue and prevent the capsule from moving around once it has been implanted.

**ANTENNA**
The coils of the antenna turn the reader’s varying magnetic field into current to power the chip. The coil is coupled to a capacitor to form a circuit that resonates at 134 kilohertz.

**ID CHIP**
The chip modulates the amplitude of the current going through the antenna to continuously repeat a 128-bit signal. The bits are represented by a change in amplitude—low to high or high to low. An analysis by Jonathan Westhues, of Cambridge, Mass., indicated that only 32 of the bits varied between any two VeriChips. The rest of the bits probably tell the reader when the loop starts and may also contain some error-checking or correction data.
Some Example of Multi-scale Problems Cont.

Trace width: 20 µm
Spacing: 15 µm

\( N = 102 \) turns (in two layers)

\( Q \approx 9 @ 2.64 \text{ MHz} \)

Images courtesy
Fraunhofer Institut Zuverlässigkeit und Mikrointegration (IZM), Berlin

http://www.iat.uzh.ch/icat/home.html
Advanced Extra Vehicular Activity (AEVA) Space Suits

CEV Launch, Return and Contingency EVA Suit

Flight Suit  In-Space Suit  Surface Suit
A Plethora of Applications

Ziolkowski’s group: resonant sub-\( \lambda \) dipole antennas

Roma Tre group: resonant sub-\( \lambda \) patch and leaky wave antennas

F. Bilotti – Potential Applications of Matamaterials in Antennas
A CNT is a rolled-up graphene sheet in which the edges of the sheet are joined together to form a seamless tube. A graphene sheet is a single layer of carbon atoms with a 2-D honeycomb lattice structure.
A cartoon drawing of the titania nanotube dye-sensitized solar cell.
METAMATERIAL DEVICES
Variety of CEM Methods

Computational methods are often preferred because of their versatile nature. Most of the commercial software in computational electromagnetics are based on the following three techniques, namely:

- **MoM**: The MoM algorithm formulates integral equations via the use of Green’s function. Since it’s a frequency domain technique, MoM can easily handle dispersive media.

- **FDTD**: The FDTD algorithm solves the difference form of the Maxwell’s differential equations. The most salient feature of this time domain technique is that it is highly parallelizable.

- **FEM**: FEM algorithm solves the differential equations using a weighted residual method. As in case of the MoM, FEM is also a frequency domain technique and, hence, can handle dispersive media with ease.

\[
\nabla \times E = -\frac{\partial B}{\partial t} \\
\nabla \times H = J_s + \frac{\partial D}{\partial t} \\
\n\nabla \cdot D = \rho \\
\n\nabla \cdot B = 0
\]
Motivation: Why do we need a New Approach?

Partial list of difficulties associated with the existing MoM formulation, which is based on using the Green’s to represent the E-field in terms of the induced current J:

a. Dealing with Singular and Hypersingular behavior of Green’s Function

\[ E = -j\omega A - \frac{j}{\omega \gamma} \nabla (\nabla \cdot A) \]

b. Handling Thin wires and/or Sheets with finite losses
Motivation (Contd.)

d. Deriving a universal approach for PEC (Surface formulation) and dielectric bodies (Volume Integral Equation)

e. Accurately modeling multi-scale geometries
f. Dealing with low-frequency breakdown

Matching Surface $d/2 \approx 0\lambda$ @ low frequencies

Current along the axis

A PEC Rod Modeled Using MoM
Motivation (Contd.)

Also, the conventional time domain technique, FDTD, demands extensive computational resources when solving low frequency problems or when dealing with dispersive media.

Reflections from PML
Some difficulties that different numerical codes have for modeling small objects

Small Wire – length \( \lambda/20 \) (Feko)

Solution from surface meshing

Scattering by a small (diam = \( \frac{13\lambda_0}{800} \)) dielectric sphere (CST)

\[ \text{H}_\phi \text{ (i.e. Jz) along circumference} \]

\[ |\text{H}(\phi)| \text{ i.e. } |\text{J}(\phi)| \]

<table>
<thead>
<tr>
<th>Magnetic field [pA/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.0</td>
</tr>
<tr>
<td>40.0</td>
</tr>
<tr>
<td>45.0</td>
</tr>
<tr>
<td>50.0</td>
</tr>
<tr>
<td>55.0</td>
</tr>
<tr>
<td>60.0</td>
</tr>
<tr>
<td>65.0</td>
</tr>
<tr>
<td>70.0</td>
</tr>
</tbody>
</table>

Position z [mm]

38
Large Vivaldi Antenna Array for Radio Astronomy

Since the building cost of the large, complex antenna array is very high, its design stage becomes extremely important and requires an accurate and efficient EM solver.

- **FDTD Model of 8 × 9 × 2 Vivaldi Array**
- **Actual Structure**
- **Job Statistics**
  - No. of FDTD cells: 1330Δ × 1330Δ × 718Δ
  - Degrees of freedom: 7.6E9
  - Computer resources: 504 CPUs on LOFAR
  - Total simulation time: <100 hrs.
  - Est. simulation time on 1 CPU with sufficient memory: ~ 2 years

- **Simulation vs. Measured Results**
  - E-plane pattern at 4.7 GHz
  - Active element input impedance real part

- **Photo of the Westerbork Synthesis Radio Telescope**
- **One Row of Vivaldi Elements**
Simulation of Metamaterials

Metamaterials are man-made composite structures that are tailored to exhibit certain unusual properties not readily observed in the constituent materials. It is both important and necessary, in real-world applications, to perform a rigorous simulation of the practical system, where the inclusions inside the metamaterial structures are also modeled accurately, in order to evaluate, in a reliable manner, the performance of a system that contains such materials.

However, it is not an easy task because of the enormous degrees of freedom involved in such simulation.

FDTD Model of Metamaterial Slab

3876 elements

Job Statistics

- Incident beam direction
- Homogeneous DNG slab with n=-1
- Ey phase
- Ey magnitude

<table>
<thead>
<tr>
<th>No. of FDTD cells</th>
<th>Degrees of freedom</th>
<th>Computer resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>684Δ x 680Δ x 582Δ</td>
<td>1.6E9</td>
<td>15 CPUs on GEMS box</td>
</tr>
</tbody>
</table>

Field Distribution on the Vertical Plane at 15.3 GHz (n = -1)

- Ey magnitude
- Ey phase

- Computer resources
- Total simulation time
- Est. simulation time on 1 CPU with sufficient memory
- ~5 days
100x100 element array computational domain

Computation domain size = (642.4, 728.625, 100.2) cm
100×100 patch array, periodicities = (6.38, 7.25) cm, Patch size = (4.13, 5.0) cm
Frequency range: 1.6 ~ 2.0 GHz

Est. Run Time on a single PC—180 days
22 mins. on the LOFAR Blue Gene cluster
Dipole Moment Approach
The scattered field from a sphere can be represented in terms of Electric and Magnetic Dipole Moments, for PEC as:

\[ \text{DM}_E = E_0 \frac{4\pi j}{\eta k^2} (k\alpha)^3 \]
\[ \text{DM}_M = E_0 \frac{2\pi}{j k^2} (k\alpha)^3 \]

And for dielectrics as:

\[ \text{DM}_E = E_0 \frac{4\pi j}{\eta k^2} (k\alpha)^3 \left( \frac{\varepsilon_r - 1}{\varepsilon_r + 2} \right) \]
\[ \text{DM}_M = E_0 \frac{4\pi j}{k^2} (k\alpha)^3 \left( \frac{\mu_r - 1}{\mu_r + 2} \right) \]
Steps involved in formulating PEC regions are:

i. Replace the original scatterer using PEC spheres.
ii. Replace these spheres using their corresponding DMs.
iii. Represent these DMs in terms of a suitable set of basis functions.
iv. Apply *boundary condition* on the tangential E-field, derived from the DMs, with a set of testing functions.
Dielectric Formulation

Steps involved in formulating dielectric regions are:

i. Replace the original scatterer using dielectric spheres.
ii. Replace these spheres using their corresponding DMs.
iii. Represent these DMs in terms of a suitable set of basis functions.
iv. Apply \textit{consistency condition} on the tangential E-field, derived from the DMs, with a set of testing functions as:

\[ \varepsilon_o (\varepsilon_r - 1)(E_{inc} + E_{scat}) = F(II) \]

Where \( F \), called as the consistency factor, is found to be:

\[ F = \frac{-3j}{4\pi\omega a^3} \]
PEC Sphere

$\lambda_o @ 10 \text{ GHz}$

Observe Point: $\lambda_o / 46$

Can be easily extended to objects with finite conductivity
PEC Sphere (Contd.)

Amplitude Comparison of $E_z$

Distance along $X$ in $\lambda$

Amplitude in V/m

Approach | No. of Unknowns
--- | ---
DM without macro-basis | 2322
DM with macro-basis | 43
Dielectric Plate

$\varepsilon_r = 6$

$\lambda/400$

$\lambda/40$

$E_y$

$k_z$

<table>
<thead>
<tr>
<th>Approach</th>
<th>No. of Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM only</td>
<td>162</td>
</tr>
<tr>
<td>DM using macro-basis</td>
<td>42</td>
</tr>
</tbody>
</table>
Dielectric Plate (Contd.)

### Amplitude Comparison

- **E_y Amplitude Comparison**
  - **DM Approach**
  - **FEKO**

### Phase Comparison

- **E_y Phase Comparison**
  - **DM Approach**
  - **FEKO**

<table>
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</table>
Numerical results – back-scattered near field

PEC sheet
\( \frac{\lambda}{2} \) on the side
\( f = 5 \text{ GHz} \)
\( r = \lambda \)

Magnitude

Phase

\( |E_s| \theta \) (deg)

\( \angle E_s \)
Numerical results – RCS

PEC sheet  $3\lambda$ on the side  $f = 1$ GHz

Bistatic RCS - Azimuth

![Graph showing bistatic RCS and azimuth](image-url)
Impedance matrix filling-time performance

PEC sheet
n^unknowns varies

Z matrix filling time
Relative time (%)

Relative time (%) is calculated as \((T_{\text{our method}}/T_{\text{conv MoM}})\times 100\)
Numerical results – RCS

PEC Corner Reflector
\( \lambda/2 \) on the side \( f = 5 \text{GHz} \)

**Bistatic RCS – Elevation \((\phi=0)\)**

**Bistatic RCS – Azimuth**
RCS performance vs n^ radiating dipoles

PEC cube

λ on the side

f = 1 GHz

| r | ≤ 0.2λ

else
RCS performance vs n^ radiating dipoles

Bistatic RCS – Elevation (φ=0)
RCS performance - dielectric

Dielectric ($\varepsilon_r = 4$) cube

$\lambda/4$ on the side

$f = 5 \text{ GHz}$

Bistatic RCS – Elevation ($\phi=0$)  

Bistatic RCS - Azimuth
Low-frequency performance – PEC sheet

PEC sheet
\( \lambda \) on the side @ 1 GHz

Z matrices condition numbers
Low-frequency performance – PEC sphere

**PEC sphere**
diameter $= \lambda$ @ 1 GHz

*Z matrices condition numbers*
Low-frequency performance – PEC sphere
Low-frequency performance – PEC sphere

- 1 MHz
- 100 KHz
- 10 KHz
OBSERVATIONS

- An efficient procedure to calculate the matrix elements from low to microwave frequencies has been shown.

- The proposed technique, bypassing the use of the potential formulation, agrees well with Mie series at all frequencies, while the conventional MoM result becomes inaccurate at lower frequencies (when its impedance matrix starts to be ill-conditioned).

- A considerable time saving is achieved in comparison with the conventional MoM, since the numerical treatment of the Green’s functions singularities is avoided.

Future Work

- Implementation of higher order Characteristic Basis Function Method (CBFM)
Recursive Update in Frequency Domain

RUFD

And

Universal FDTD

For Time and Frequency Domains

Without

Matrix Inversion or Iterative Solution
Maxwell’s Equations in Frequency Domain

\[ j\omega \hat{E} = \frac{1}{\varepsilon} \nabla \times \hat{H} - \frac{\sigma}{\varepsilon} \hat{E}, \]

\[ j\omega \hat{H} = -\frac{1}{\mu} \nabla \times \hat{E} - \frac{\sigma^*}{\mu} \hat{H}, \]

Typically, the FEM or FDFD algorithms are used in the frequency domain. A matrix inversion is required for solution. But often the matrix is found to be ill-conditioned and requires sophisticated preconditioners.
Maxwell’s Equations in Time Domain

\[
\begin{align*}
\frac{\partial \vec{E}}{\partial t} &= \frac{1}{\varepsilon} \nabla \times \hat{H} - \frac{\sigma}{\varepsilon} \hat{E}, \\
\frac{\partial \vec{H}}{\partial t} &= -\frac{1}{\mu} \nabla \times \hat{E} - \frac{\sigma^*}{\mu} \hat{H},
\end{align*}
\]

\( E^{n+1}_x \left(i - \frac{1}{2}, j, k\right) \)

\[= E^n_x \left(i - \frac{1}{2}, j, k\right) + \frac{\kappa t}{\Delta y} [H^{n+\frac{1}{2}}_x \left(i - \frac{1}{2}, j + \frac{1}{2}, k\right) - H^{n-\frac{1}{2}}_x \left(i - \frac{1}{2}, j - \frac{1}{2}, k\right)] - \frac{\kappa t}{\Delta y} [H^{n+\frac{1}{2}}_y \left(i - \frac{1}{2}, j, k + \frac{1}{2}\right) - H^{n-\frac{1}{2}}_y \left(i - \frac{1}{2}, j, k - \frac{1}{2}\right)] \]

We solve the time domain equation using the leapfrog algorithm FDTD, which is a recursive scheme that requires no matrix inversion. But FDTD demands extensive computational resources when solving low frequency problems or when dealing with dispersive media.
RUFD: Recursive Update in Frequency Domain

Apply FDTD-discretization and use leap-frog algorithm tailored for the frequency domain rather than solve a matrix equation

\[
e^{j\omega \tau} \hat{E}^{n+1} - \hat{E}^n = \frac{1}{\tau} \nabla \times \hat{H}^{n+1/2} e^{j\omega \tau/2} - \frac{\sigma}{\varepsilon} e^{j\omega \tau} \hat{E}^{n+1}
\]

\[
e^{j\omega \tau/2} \hat{H}^{n+1/2} - e^{-j\omega \tau/2} \hat{H}^{n-1/2} = -\frac{1}{\mu} \nabla \times \hat{E}^n - \frac{\sigma^*}{\mu} \hat{H}^{n+1/2} e^{j\omega \tau/2}
\]

Where

\[
\hat{E}^n = E^n e^{j\omega \tau}
\]

\[
\hat{H}^{n+1/2} = H^{n+1/2} e^{j\omega (n+\frac{1}{2})\tau}
\]
Relationship between RUFD and Maxwell’s Equations

\[
\frac{e^{j\omega \tau} \hat{E}^{n+1} - \hat{E}^n}{\tau} = \frac{1}{\varepsilon} \nabla \times \hat{H}^{n+1/2} e^{j\omega \tau/2} - \frac{\sigma}{\varepsilon} e^{j\omega \tau} \hat{E}^{n+1}
\]

\[
\tau \to 0
\]

\[
j\omega \hat{E} = \frac{1}{\varepsilon} \nabla \times \hat{H} - \frac{\sigma}{\varepsilon} \hat{E}
\]

Provided \( \frac{\tau}{\hbar} \leq \sqrt{\frac{\varepsilon \mu}{8}} \) is satisfied to maintain stability
PEC Plate

![Diagram of PEC Plate]

**Total E\textsubscript{z} Comparison**

- T/S Formulation
- S Formulation
- FEKO

**Along Y**

- **Amplitude in V/m**
  - 0
  - 0.5
  - 1
  - 1.5
  - 2
  - 2.5

- **Phase in Degrees**
  - -100
  - -50
  - 0
  - 50
  - 100
  - 150

\(\lambda \text{ @ 10 GHz}\)
Dielectric Cube

$\lambda @ 10$ GHz

$\varepsilon_r = 6$

Amplitude variation of Backscattered $E_z$

Phase variation of Backscattered $E_z$
Dipole Antenna

Feed Gap Source $E_z$ ($\lambda/20$)

$\lambda$ @ 10 GHz
Dipole Antenna Contd.

Variation of Input Resistance

Variation of Input Reactance

73 Ω @ 8.45 GHz

j0 Ω @ 8.45 GHz

Resistance

Reactance
Observations

Some of the advantages of the proposed method are:

I. Since RUFD is a frequency domain solver, it handles dispersive media more conveniently than a time domain solver.

II. Since RUFD solves Maxwell’s equation recursively, instead of using matrix inversion, the problems of ill-conditioned matrices and the need for constructing robust pre-conditioners are totally avoided.

III. Efficient for constructing low frequency solution, compared to the long runs in FDTD.

IV. The recursive update equations used in RUFD algorithm are highly parallelizable.

V. The use of Yee’s cell makes the meshing relatively simple.
Enhancement of RUFD

Enhance DM Approach by hybridizing it with RUFD. The goals of this research are to:

I. Demonstrate the use of the DM Approach for problems involving slits and slots.
CHARACTERISTIC BASIS FUNCTION METHOD (CBFM)
DOMAIN DECOMPOSITION & PARALLELIZATION IN MoM

CHARACTERISTIC BASIS FUNCTION METHOD
CBFM
**Radiation Problem**

- Simulation has been carried by using Locally Modified Approach.
- Only the block, where the antenna is mounted on, has been analyzed.
- Parametric analysis can be useful to locate the best antenna position.

### Antenna Geometrical Description

<table>
<thead>
<tr>
<th>Height</th>
<th>Meter</th>
<th>Wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.375</td>
<td>0.375</td>
<td>λ/4</td>
</tr>
</tbody>
</table>

Time Performance drops off ~ 80%

![λ/4 Monopole Antenna](image)
Block decomposition

300MHz

Level #1

Level #2
Fig. 5: Array of 49 subarrays (7x7), each of them composed of 9 TSA elements (3x3). To illustrate coupling effects, the active antennas within the central tile are excited by a voltage-gap generator placed over the slot of each TSA element. The central tile scans to broadside (end-fire direction), whereas the TSAs of the surrounding tiles are short-circuited. The magnitude of the surface current distribution is shown (log scale) as computed by a direct CBFM approach.
Enhancement (Contd.)

II. Use of MBFs, for which closed form field expressions are readily available in order to speed up the matrix generation.

III. Hybridization of the DM Approach with RUFD algorithm in order to solve multi-scale problem with relative ease and accuracy.
Problems with Multi-scale geometries in CEM

- For finite methods, the computational domain size can be electrically large (several wavelengths), whereas many of the objects within the domain possess fine features (small fraction of the operating wavelength).

- In MoM fine features can cause the matrix condition number to be poor.

- Major bottleneck in terms of memory and runtime requirements.

- Many practical problems are multi-scale, and these continue to push current numerical approaches to their limit.

**Arbitrarily Oriented Thin conducting wire, with radius r<<λ**

- Represent conductors using DMs
- Pass the scattered field info to the FDTD whose cell size remains unchanged (say Lambda/20)
- Let FDTD propagate the scattered field in the computational domain (other objects may be present, i.e. ground planes)

![Diagram of FDTD process](image)
Scattering by Two Dielectric Spheres:

The two dielectric spheres are $\lambda/200$ thick. They are placed at $\pm \lambda/40$ along $Y$. The fields are measured along $Z$ passing through $Y = -\lambda/40$, from $\lambda/20$ to $\lambda$. Frequency of interest is 300 MHz. $\varepsilon_r = 6$.

Scattering by a Slanted PEC Wire:

wire with length $\lambda/30$ and wire radius $\lambda/200$
A dispersive sphere with $\varepsilon_r = -47.5378 - 1.1383j$ (Gold) is placed in free space ($\lambda$ @ 300THz). Here the results from DM Approach are compared with HFSS. The fields are measured along $Z$ from $-\lambda/2$ to $\lambda/2$. 
$E_y$ Variation (Along Z)

**Amplitude Variation of Scattered $E_y$**
- DM Approach
- HFSS

**Phase Variation of Scattered $E_y$**
- DM Approach
- HFSS

Amplitude in V/m

Phase in Degrees

Distance along Z in $\lambda$
Scattering by Thin Wire Helix in Two FDTD Cells:
Scattering by a Coated PEC Wire in Two FDTD Cells:

Coated PEC wire: inner conductor radius $\frac{\lambda_0}{200}$, coating thickness $\delta = \frac{\lambda_0}{80}$

Scattering by a Small PEC loop:

PEC loop with radius $\frac{\lambda_0}{20}$
Short Monopole in Transmit mode:

*For transmitting cases the Hybrid DM/FDTD scheme requires a calibration factor to couple to the FDTD.
Total fields (RUFD TF/SF formulation)

Ez comparison along y axis varying the mesh

Magnitude

Phase

Ez, total - Magnitude -

Ez, total - Phase -

Mar 16 2010
### Performance Summary

<table>
<thead>
<tr>
<th>Memory (Peak)</th>
<th>RUFD</th>
<th>HFSS</th>
<th>FEKO</th>
<th>CST</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda/20 )</td>
<td>297 MB</td>
<td>266 MB</td>
<td>281.5 MB</td>
<td>689.83 MB</td>
</tr>
<tr>
<td>( dy = \lambda/20 ) ( dx = dz = \lambda/10 )</td>
<td>263 MB</td>
<td></td>
<td></td>
<td>(ph: 499.54MB vir: 770.13MB)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>RUFD</th>
<th>HFSS</th>
<th>FEKO</th>
<th>CST</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda/20 )</td>
<td>158.4 s</td>
<td>57 s</td>
<td>268 s</td>
<td></td>
</tr>
<tr>
<td>( dy = \lambda/20 ) ( dx = dz = \lambda/10 )</td>
<td>29.46 s</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>( \lambda/10 )</td>
<td>13.81 s</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

RUFD: 1000 its, DFT after 500
FEKO: edge length \( \lambda/10 - \lambda/20 \)

*Accuracy is poor
Self Consistent Hybridization - New Procedure

- We run the $\lambda/2$ PEC wire ($\lambda/80$ in diameter) and $\lambda$ on the side dielectric coated ($\varepsilon_r = 6$) ($\lambda/10$ thick) square PEC sheet case by hybridizing
  - Jordan method with RUFD with New Self Consistent procedure
- Cellsize in RUFD is $\lambda/20$, we run 500 its and do the DFT after 250
- We compare the Total/Scattered Fields from the hybrid technique with HFSS/FEKO/CST
\[ \mathbf{E}_{s,f} = x \mathbf{E}_{s, \text{RUFD/SFF(1)}} + \mathbf{E}_{s, \text{RUFD (PW)}} + x \mathbf{E}_{s, \text{RUFD/SFF(2)}} \]

Fields produced by RUFD/SFF(1) by using the fields scattered by a unit amplitude current as a Hard Source

RUFD isolated problem for PW incidence \( \mathbf{E}_s \)

Fields produced by last RUFD/SFF step

**Magnitude**

Amplitude of \( \mathbf{E}_s \)

- **Es final**
- **HFSS**
- **FEKO**

**Phase**

Phase of \( \mathbf{E}_s \)

- **Es final**
- **HFSS**
- **FEKO**

Apr 14 2010
Performance Summary

<table>
<thead>
<tr>
<th>Hybrid J/RUFD</th>
<th>FEKO</th>
<th>CST (FD)</th>
<th>HFSS</th>
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</thead>
<tbody>
<tr>
<td>dx=dy=dz=λ/20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Memory (Peak)</td>
<td>348 MB</td>
<td>690.6 MB</td>
<td>ph: 1.59 GB</td>
</tr>
<tr>
<td>Time</td>
<td>260 s</td>
<td>284.12 s</td>
<td>1565 s</td>
</tr>
</tbody>
</table>

Simulations have been carried out on 4GB RAM and 3 GHz intel core 2Duo processor

- **FEKO**: surface edge length for the wire: λ/100; for the plate: λ/20 – (Matrix condition number: 7.21E+04 – some warnings appeared: ‘singular field on the surface of a triangle’). (NOT ACCURATE)
- **HFSS**: 4 adaptive mesh refinement passes requested, with a threshold of 0.01 (6 passes done) – Mesh started from 36170 tetrahedra
- **CST**: The mesh has been manually increased for the wire before the simulation started – 8 adaptive mesh refinement passes done
A Multilevel Characteristic Basis Finite Element Method for Efficient Solution of Large EM Problems

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Overview of Characteristic Basis Finite Element Method (CBFEM)

Versions:
- CBFEM-MPI: CBFEM for quasi-static problems
- CBFEM-MPI: Dipole-based CBFEM for EM problems
- CBFEM-PO: Physical Optics-based CBFEM for EM problems

- ML-CBFEM: Multi-level CBFEM
  (next generation CBFEM)
  Built on any CBFEM approach above

Numerical Examples (3D scattering problems)
CBFs (Characteristic Basis Functions)
“macro-domain basis functions that are tailored to take the physics of the problem into account”
Parallel, non-iterative DD algorithm

\[
[U_j]_{N_j \times K_j} = \left( [u_j^{(1)}], [u_j^{(2)}], \ldots, [u_j^{(K_j)}] \right)
\]

\[
[e_j]_{N_j \times 1} = \sum_{i=1}^{K_j} c_j^{(i)} [u_j^{(i)}]_{N_j \times 1}
\]

\[
[S_{\alpha \beta}] = [U_\alpha]^T [A_{\alpha \beta}] [U_\beta]
\]
Locate fictitious “point charges” on the conductors to generate CBFs

\[ \nabla \cdot [\varepsilon \nabla \phi(\vec{r})] = 0 \]

BCs: \( \phi(\vec{r}) = 0 \) on the outer box

\( \phi(\vec{r}) = 1 \) on the excited conductor(s)


*Microwave and Optical Technology Letters*, vol.52, no. 5, pp. 1053-1060, May 2010.
First Level:
• Create CBFs for each interface.
• Create CBFs for each sub-domain by solving each sub-problem by using interface bases as the excitation functions.

Higher Levels:
• Group a number of sub-domains belonging to the first level, and employ their bases to find the bases of the second level.
• Progressively combine the bases by grouping a number of sub-domains into larger sub-domains of the higher levels.
• Derive the CBFs in the j-th sub-domain of the (m+1)th level by expressing the bases as a linear combination of the bases in the m-th level in terms of a set of coefficients.

\[
\sum_{i=1}^{K_j} C_i \begin{bmatrix} u_i^{(m)} \end{bmatrix} \]
NUMERICAL RESULTS
(ML-CBFEM with PO approach)
Level # 1: 40 sub-domains
Level # 2: 20 sub-domains

\( \theta_{\text{inc}}: [0^\circ, 20^\circ, \ldots, 120^\circ, 150^\circ, 180^\circ] \)
\( \phi_{\text{inc}}: [0^\circ, 90^\circ, 180^\circ, 270^\circ] \)
\( \theta_{-\text{polrz}} & \phi_{-\text{polrz}} \)

\[ k = [6.2832 \text{ (actual), } 1\ldots5, 7\ldots22] \]
Center freq = 300 Mhz
Min. freq = 48 Mhz
Max. freq = 1 GHz

<table>
<thead>
<tr>
<th>Original matrix size</th>
<th>1,446,447</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduced matrix size (Level 1)</td>
<td>28,886</td>
</tr>
<tr>
<td>Reduced matrix size (Level 2)</td>
<td>14,087</td>
</tr>
</tbody>
</table>

**Length** 31.5 \( \lambda \)
**Radome Diameter** 3 \( \lambda \)
Plate

Level #1: 52 sub-domains
Level #2: 26 sub-domains

---

Max. physical memory

<table>
<thead>
<tr>
<th>Level</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>28 GB</td>
</tr>
<tr>
<td>Level 2</td>
<td>8 GB</td>
</tr>
</tbody>
</table>

---

Original matrix size : 1,800,205
Reduced matrix size (Level 1) : 32,958
Reduced matrix size (Level 2) : 16,130

---

Bistatic RCS Profile ($\phi=0^\circ$ plane)

Level #1: 52 sub-domains
Level #2: 26 sub-domains

---

RCS / $\lambda^2$ (dB)

---

$E^{inc}$

---

(a)

---

(b)

---

(c)
Multi-sphere problem

Level # 1

Original matrix size : 198,677
Reduced matrix size (Level 1) : 2,285
Reduced matrix size (Level 2) : 743
Reduced matrix size (Level 3) : 248

Bistatic RCS Profile ($\delta=0^\circ$ plane)

- Conv. FEM
- Conv. CBFM (level-1)
- ML-CBFM (level-2)
- ML-CBFM (level-3)

Apr 14 2010
DEPARTURES FROM CONVENTIONAL CEM FORMULATIONS

1. Formulating MoM problems w/o Green’s functions—
   universal for different materials, homogeneous or
   inhomogeneous, lossy or lossless, thin or thick (no
   restriction), and valid for all frequencies including low
   frequencies (no singularity problems, no need for loop-star
   basis functions), no need for impedance boundary conditions

2. Layered medium problem without Sommerfeld Integrals

3. Periodic Structures without Periodic Green’s Functions

4. Solving Large MoM and FEM problems without Iteration by
   using CBFM (highly Parallelizable formulation, easy
   handling of Multiple rhs), and EFIE even for closed bodies

5. RUFD: New general-purpose and highly parallelizable Finite
   Method which is both Matrix-free and iteration-free; no low
   frequency limitations

6. Hybridizing DM with RUFD and FDTD for Multiscale probs.
FAQ’S & COMMENTS

Q1. What’s wrong with using Green’s function to formulate MoM problems?
Q2. Don’t you get poorly-conditioned matrices when you use an alternate formulation, which doesn’t have to deal with the singularity in the self-term?
Q3. How can you not have problems at low frequencies and not have to use special basis functions such as loop-star, or special preconditioners?
Q4. Are you just calling the well-known Finite Difference Frequency Domain (FDFD) approach by a different name (RUFD)? Is it a smokescreen?
Q5. Have you considered wearing a bullet-proof vest?
Q6. What time is the coffee break?

C1. I don’t believe you can avoid computing Sommerfeld integrals for layered media. Who are you kidding?
C2. Never heard of anybody solving periodic structure problems without using periodic Green’s functions! Yeah, I have my doubts!
C3. There is no way you can beat FMM with CBFM, despite your claims!
C4. I just don’t believe that you are having all these problems with the commercial codes when dealing with multi-scale problems. They are gold standards! Bet you are not using them correctly!